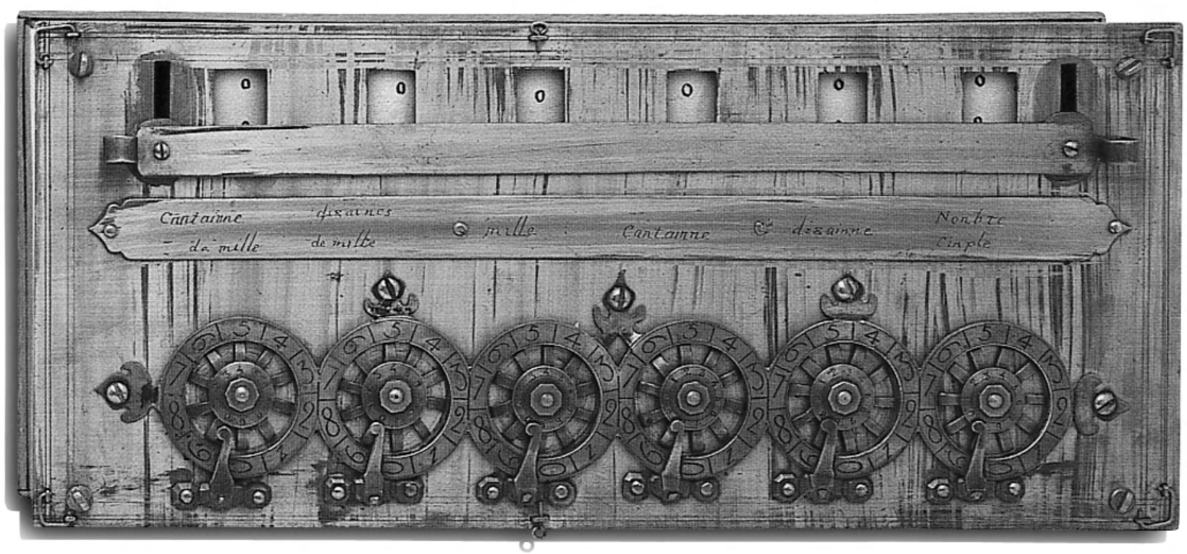
SAMPLE PAGES FOR:
PASSING THE GEORGIA HIGH SCHOOL
MATHEMATICS TEST

GPS EDITION



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CHAPTER 1 SKETCHING AND TRANSFORMING FUNCTIONS

Standards Assessed
MM1A1 b, c
➤ *Graph and identify basic functions limited to f(x) = x^n where n = 1 to 3,* $f(x)=\frac{1}{x}$, $f(x)=\sqrt{x}$, *and* $f(x)=|x|$.
➤ *Graph the basic functions, including vertical shifts, stretches, and shrinks as well as reflections across the x- and y-axes.*

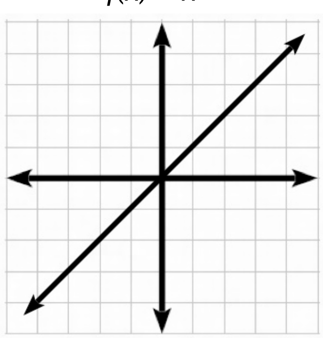
Essential Ideas
➤ All functions have a distinct shape when sketched on a coordinate plane.
➤ Functions with similar characteristics are grouped into families.
➤ Families have a **parent function**, which is the most basic shape of all functions of a family.

Vocabulary
➤ *transformation* – It changes the position, direction or shape of a function.
➤ *translation* – A translation is a transformation where the shape is not changed, but the position of the function is changed. To translate something, you slide it.
➤ *dilation* – A dilation is a transformation where the shape of the function is stretched.

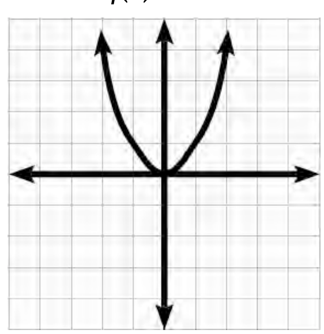
BASIC PARENT FUNCTIONS

The six basic parent functions are diagrammed below and must be memorized.

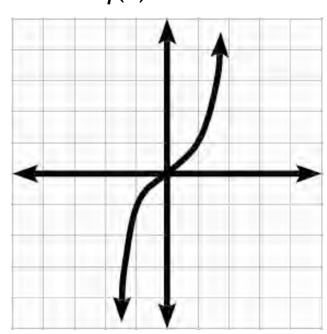
$f(x) = x$



$f(x) = x^2$



$f(x) = x^3$

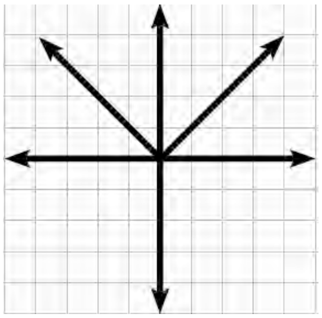


Note that all the first 5 parent functions pass through (0,0). The last function does not pass through the origin, but is symmetric about the origin.

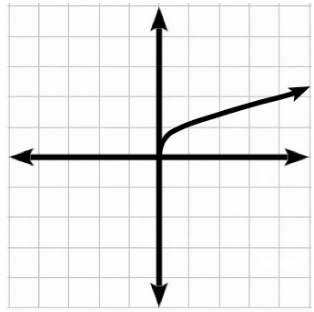
VERTICAL SHIFTS (TRANSLATIONS)

If the shape of the parent function is known, then sketching the graphs of functions becomes easier. To shift a function up or down, add or subtract a number from the parent function.

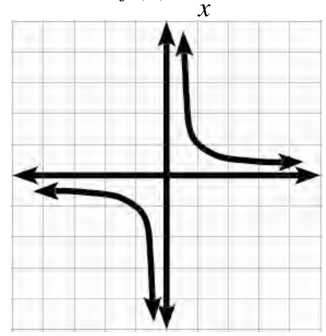
$f(x) = |x|$



$f(x) = \sqrt{x}$



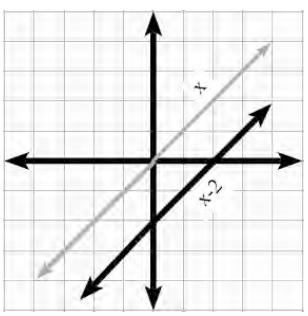
$f(x) = \frac{1}{x}$



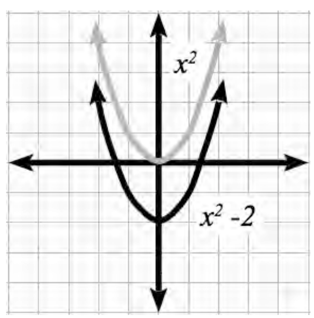
➤ **Sketch the graph of $f(x) = x - 2$, $f(x) = x^2 - 2$ and $f(x) = x^3 - 2$.**

Answer

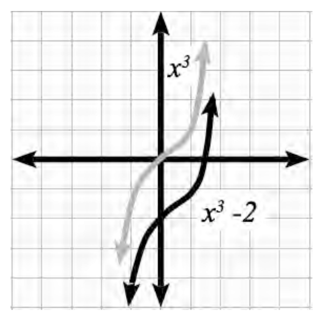
$f(x) = x - 2$



$f(x) = x^2 - 2$



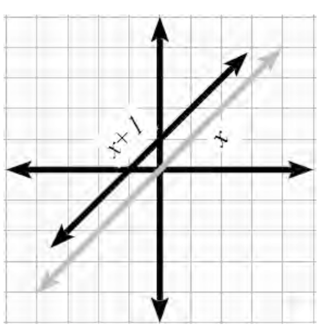
$f(x) = x^3 - 2$



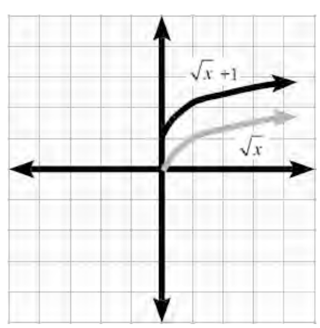
So, subtracting 2 only moves the parent function down 2. The shape of the function is not changed.

Adding a number to a function translates it up. Below, the parent functions have all been translated up 1.

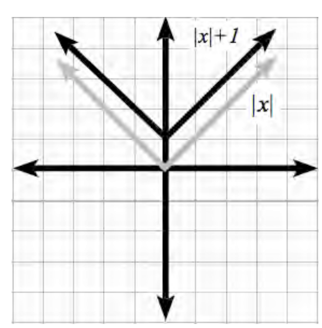
$f(x) = x$



$f(x) = \sqrt{x} + 1$



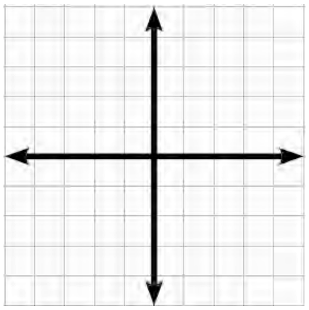
$f(x) = |x| + 1$



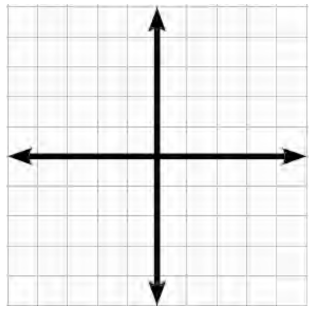
PRACTICE

Sketch each of the following.

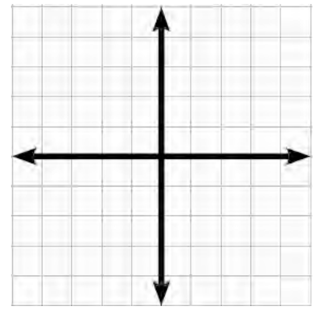
1. $f(x) = x - 3$



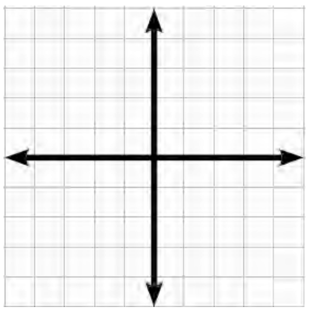
2. $f(x) = x^2 - 1$



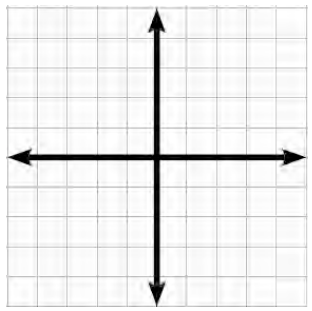
3. $f(x) = x^3 + 3$



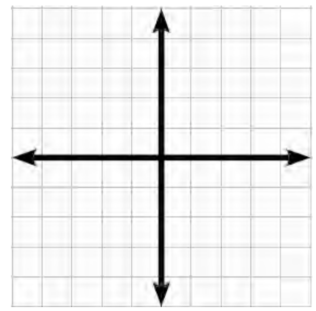
4. $f(x) = |x| - 5$



5. $f(x) = \sqrt{x} + 2$

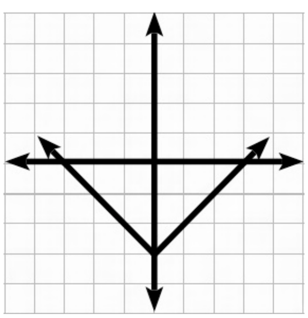


6. $f(x) = x^3 - 3$



Write a function for each.

7.________________



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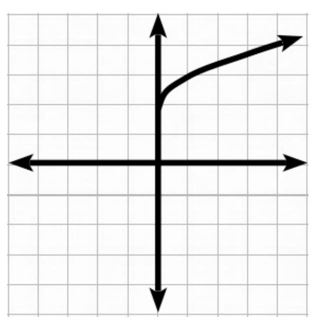

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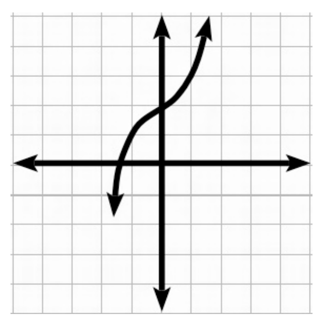


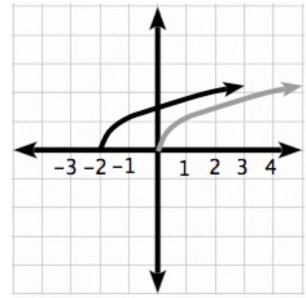
HORIZONTAL SHIFTS

To translate a parent function's graph to the left or the right, it is necessary to add or subtract the number *before* the function is evaluated.

To determine the horizontal shift, determine what number makes the part of the function inside the parentheses equal to zero. The number found represents how many spaces left or right to shift the graph.

➤ **Sketch $f(x) = (x + 2)^2$.**

What number makes $(x + 2)$ equal to zero? Or in math terms, can you solve $x + 2 = 0$? Subtracting 2 to each side gives $x = -2$. This tells you that the parent function has been shifted to the number –2 on the x–axis.

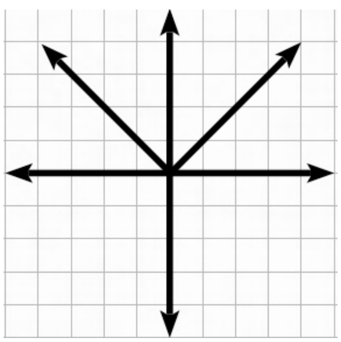


➤ **Sketch the following:**

$f(x) = |x|$
$f(x) = |x - 1|$
$f(x) = |x + 2|$

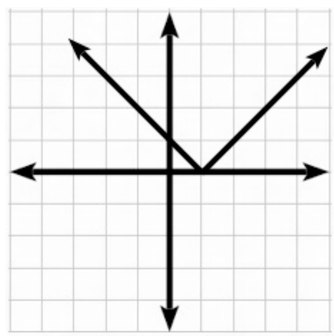
Answer

$f(x) = |x|$



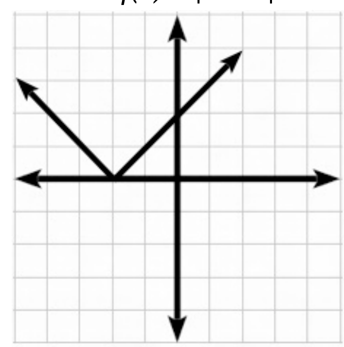
This is the parent function of absolute value.

$f(x) = |x - 1|$



What number makes $x - 1 = 0$?
Add 1 to each side $+1 \quad +1$
$x = 1$
So, the graph is shifted to 1 on the x–axis.

$f(x) = |x + 2|$

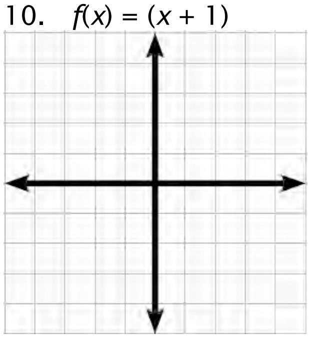


What number makes $x + 2 = 0$?
Subtract 2 on both sides $-2 \quad -2$
$x = -2$
So, the graph is shifted to –2 on the x–axis.

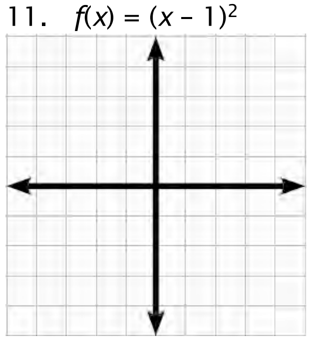
PRACTICE

Sketch each of the following.

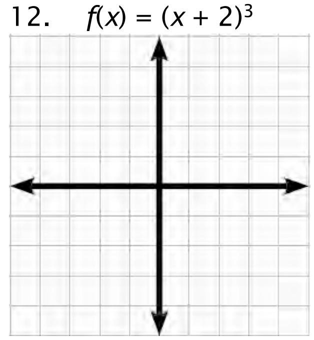
10. $f(x) = (x + 1)$



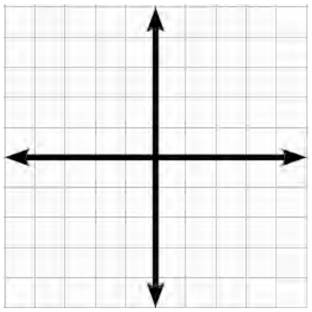
11. $f(x) = (x - 1)^2$



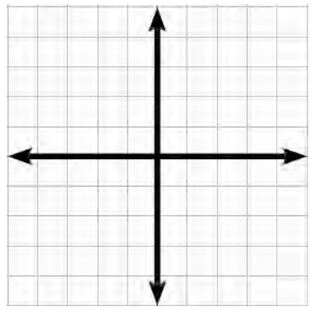
12. $f(x) = (x + 2)^3$



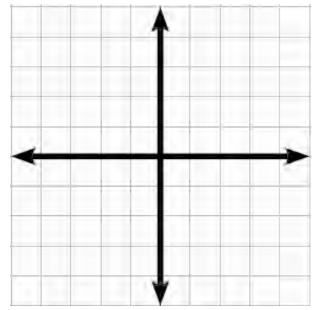
13. $f(x) = \dfrac{1}{(x-2)}$



14. $f(x) = |x + 3|$

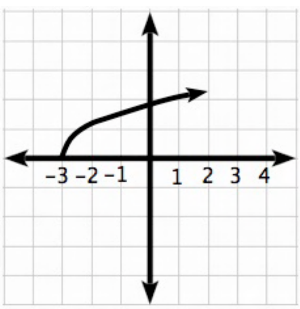


15. $f(x) = (x + 2)^2$



Write a function for each.

16.________________



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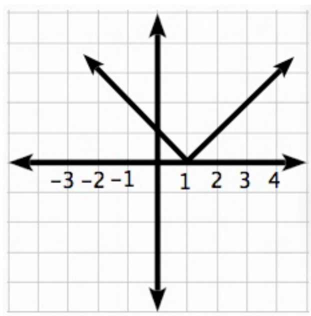

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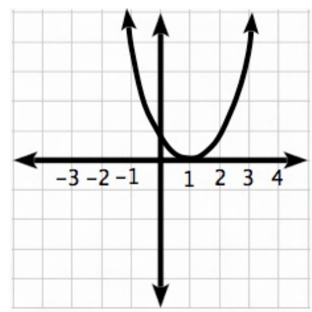
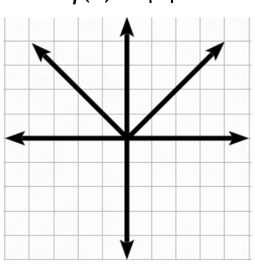


REFLECTIONS

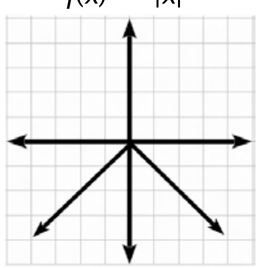
The parent functions can be reflected down over the *x*-axis by putting a negative in front of the function.

Below are the graphs of some parent functions and the functions reflected over the *x*-axis.

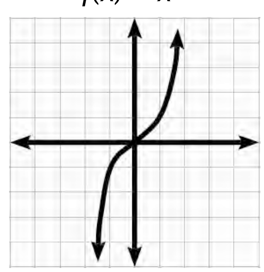
$f(x) = |x|$



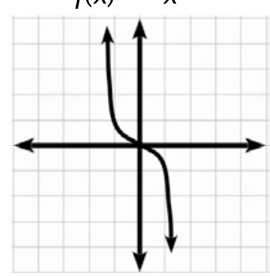
$f(x) = -|x|$



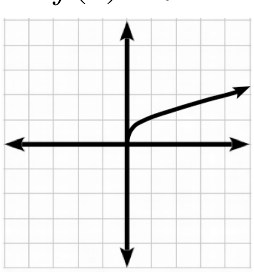
$f(x) = x^3$



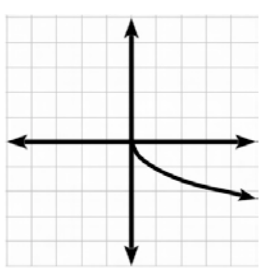
$f(x) = -x^3$



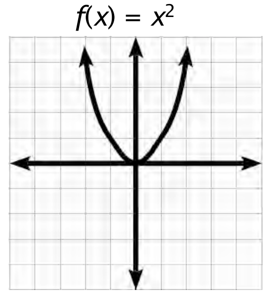
$f(x) = \sqrt{x}$



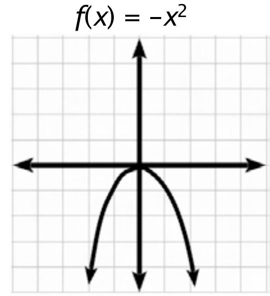
$f(x) = -\sqrt{x}$



$f(x) = x^2$

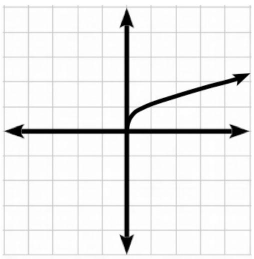


$f(x) = -x^2$

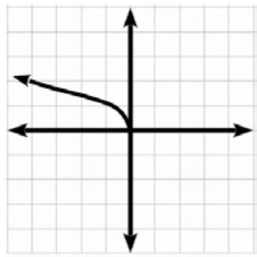


The parent function can be reflected over the *y*–axis as well. To reflect over the *y*–axis, a negative is used *inside* the function beside the variable.

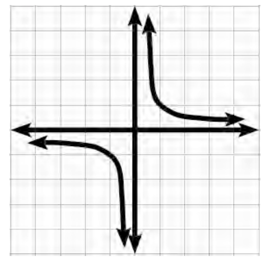
$f(x)=\sqrt{x}$



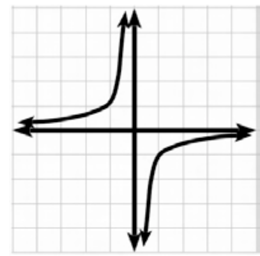
$f(x)=\sqrt{-x}$



$f(x)=\frac{1}{x}$



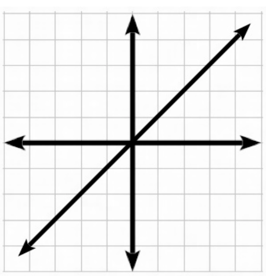
$f(x)=-\frac{1}{x}$



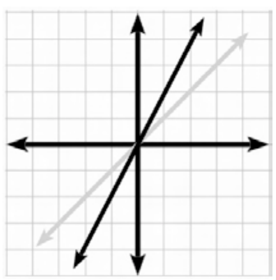
STRETCHES (DILATIONS)

A constant multiplied by a variable causes the function's shape to be stretched or shrunk vertically. A number bigger than 1 (or less than –1) multiplied to the variable causes the graph to rise faster.

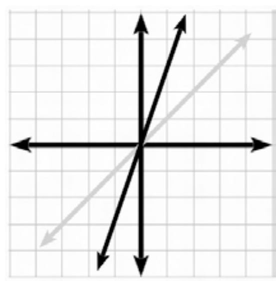
$f(x) = x$



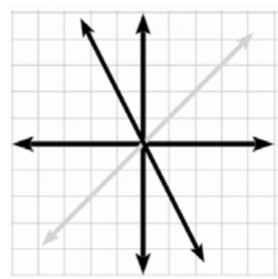
$f(x) = 2x$



$f(x) = 3x$

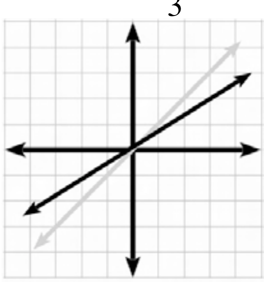


$f(x) = -2x$

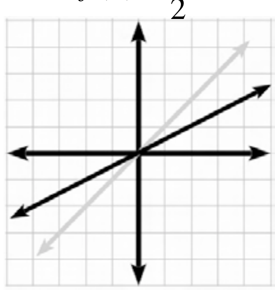


A number less than 1 causes the parent graph to rise more slowly.

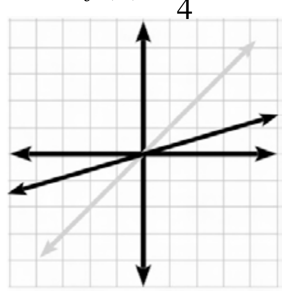
$f(x)=\frac{2}{3}x$



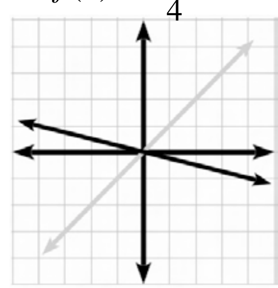
$f(x)=\frac{1}{2}x$



$f(x)=\frac{1}{4}x$

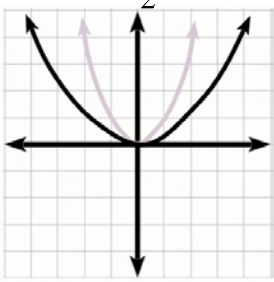


$f(x)=-\frac{1}{4}x$

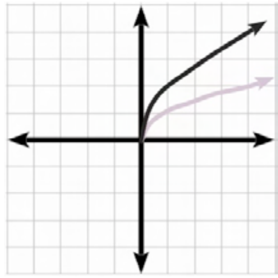


Here are some other examples.

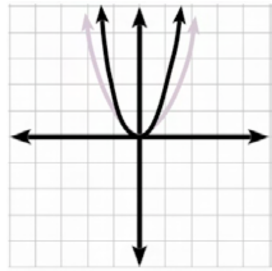
$f(x) = \frac{1}{2}x^2$



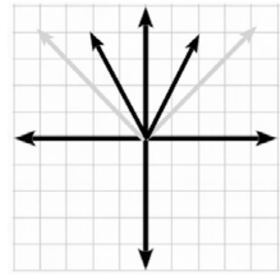
$f(x) = 2\sqrt{x}$



$f(x) = 2x^2$

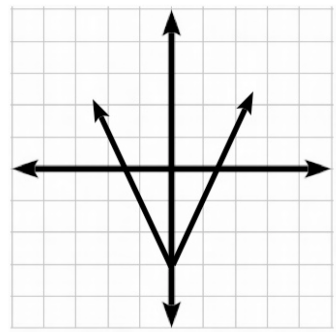


$f(x) = 2|x|$



➤ **Sketch $f(x) = 2|x| - 3$.**

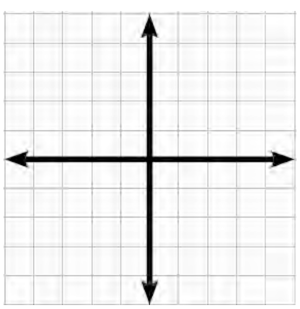
The function is $|x|$, so it will look like a V.
The 3 translates the graph down 3 spaces on the graph.
The 2 multiplied makes the function rise twice as fast.



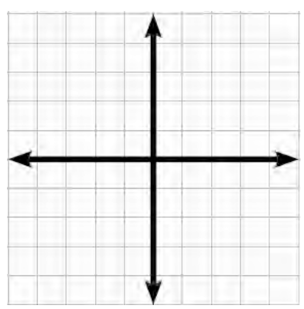
PRACTICE

Sketch these.

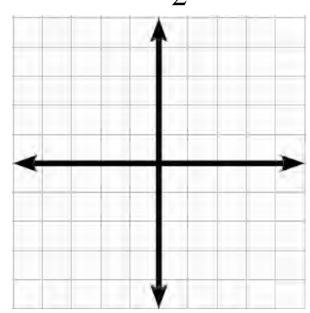
19. $f(x) = -x^2 + 1$



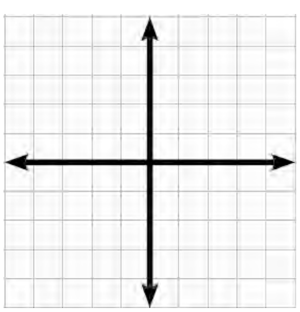
20. $f(x) = -|x - 2| + 4$



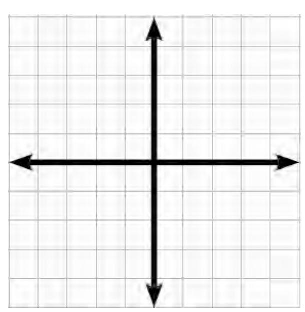
21. $f(x) = \frac{1}{2}x^3 + 1$



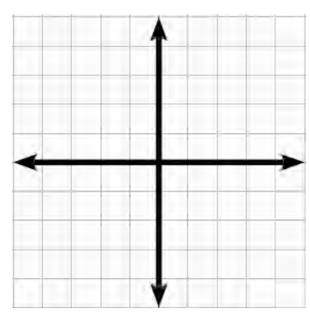
22. $f(x) = -3x + 2$



23. $f(x) = 2\sqrt{x} - 1$

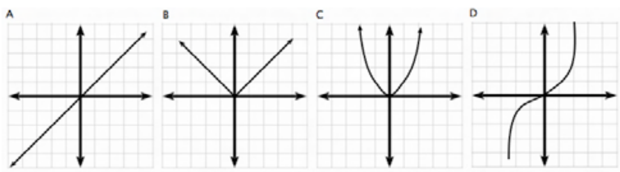


24. $f(x) = -(x+1)^2 - 2$

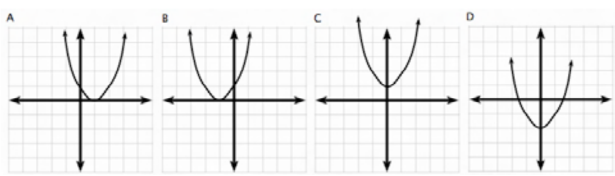


CHAPTER 1 TEST

1. Which sketch represents $f(x) = x^3$?

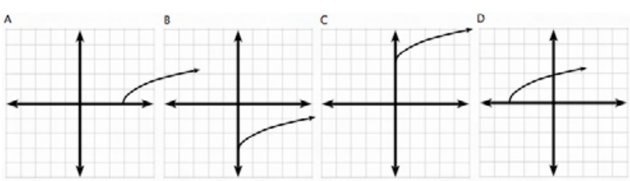


2. Which sketch represents $f(x) = x^2 + 1$?

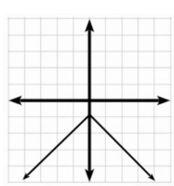


3. The function $f(x) = |x|$ is shifted up 1 and right 3. Which function below represents the transformed function?
   A. $f(x) = |x + 3| + 1$
   B. $f(x) = |x - 3| + 1$
   C. $f(x) = |x - 3| - 1$
   D. $f(x) = |x + 3| - 1$

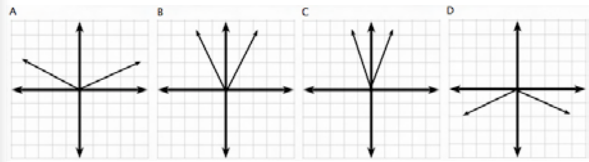
4. Which of the following is the correct sketch of $f(x) = \sqrt{x+3}$?



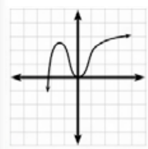
5. A function is sketched at the right. Which function represents the sketch?
   A. $f(x) = -|x| + 1$
   B. $f(x) = -|x| - 1$
   C. $f(x) = |-x| - 1$
   D. $f(x) = |x| - 1$

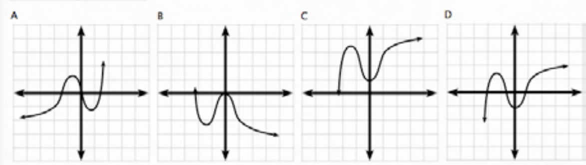


6. A function is stated in the form of $f(x) = a|x|$ where a is a variable. Which graph would have $0 < a < 1$?



7. The sketch of $f(x)$ is shown below. Which of the following would represent $-f(x)$?





8. The function $f(x) = x^2$ is reflected over the x-axis. Which of the following would be the equation of the new function?
   A. $f(x) = -x^2$
   B. $f(x) = (-x)^2$
   C. $f(x) = x^2 - 1$
   D. $f(x) = -x^2 + 1$

9. Which of the following would represent a dilation?
   A. $f(x) = -x^2$
   B. $f(x) = (-x)^2$
   C. $f(x) = 3x^2$
   D. $f(x) = (x - 3)^2$

10. Which of the following represents a translation?
   A. $f(x) = x^2 + 1$
   B. $f(x) = -x^2$
   C. $f(x) = 3x^2$
   D. $f(x) = 3(-x)^2$

CHAPTER 2 FUNCTION CHARACTERISTICS

Standards Assessed
MM1A1d, MM1A1e
➤ *Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.*
➤ *Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.*

Essential Ideas
➤ Functions can model real world problems.
➤ Knowing the characteristics of the functions will simplify identifying key parts of the functions.

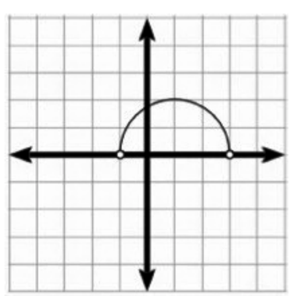
Vocabulary
➤ *domain* – The domain of a function is the set of numbers for the independent variable for which the function is defined. The domain is usually referred to as the *input values*.
➤ *range* – The range is the set of all possible *output values* of the function.

➤**Give the domain and range of the function described in the table below.**
Answer
The domain is {–2, –1, 0, 1, 2}.

x	–2	–1	0	1	2
$f(x)$	4	1	0	1	4

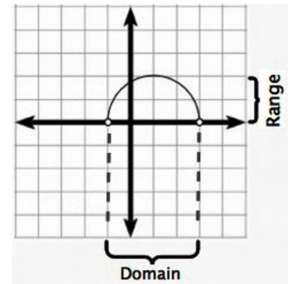
The range is {0, 1, 4}. (It is not necessary to list numbers more than once).

➤**Give the domain and range of the function sketched below.**

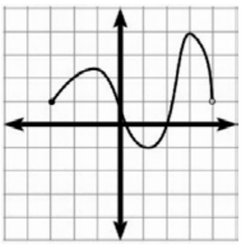


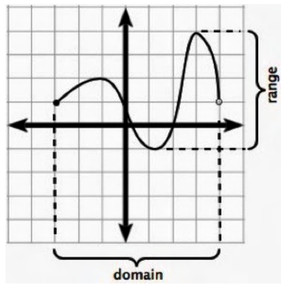
Answer
Domain: $-1 < x < 3$. The function is dispersed over all x values from –1 to 3.
Range: $0 < y \leq 2$. All of the function's y values are between 0 and 2.

The function has an open circle at (–1,0) and (3,0) These points are not included, so no equal sign is used under the inequality. Since there is no open point for the top of the semicircle, we can assume that the y values pass through the point (2, 2).



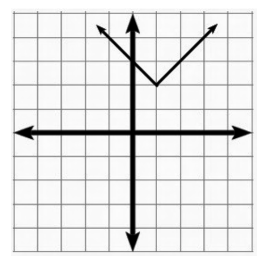
➤**Find the domain and range of the following function that is sketched below.**

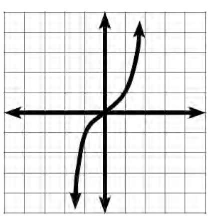




Answer
Domain: $-3 \le x < 4$
Range: $-1 \le y \le 4$
If you are asked for the domain and range of a function given its equation, then you will have to recall what the basic shape looks like.

➤**Find the domain and range of $f(x) = |x - 1| + 2$.**
Answer
Any real number can be put into the function and evaluated. So, the domain is all real numbers. The lowest y value is 2 and the function continues upward. If you are not sure, sketch a graph.
Domain: all real numbers
Range: $2 \le y < \infty$





➤**Find the domain and range of $f(x) = x^3$.**

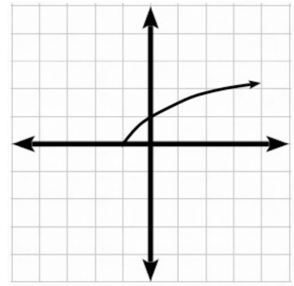
Answer
Domain: all real numbers
Range: all real numbers

Functions with domains that are not all real numbers:
- any function with a variable in the denominator
- any function with a variable inside a square root (or any other even root)

➤**Find the domain and range of** $f(x) = \dfrac{1}{(x-2)}$**.**
Answer
The domain is any number that you can put into the equation for which the output be defined. A number is undefined when the denominator equals 0. So, what number can you put into the bottom of the fraction and make it zero? If you can answer that, then you know which number is not in the domain. The number 2 makes the denominator of the fraction equal zero. Thus, it is not in the domain.
D: $x \neq 2$ or in words, all numbers except 2
R: all numbers except 0 (more on this later)

➤**Find the domain and range of $f(x) = \sqrt{x+1}$.**

Answer



For square roots, *x* values that make the square root positive are okay. Any *x* values that make the square root negative are not in the domain. The key is to determine what number makes the square root equal to zero. Then, you will have to determine which numbers make the square root positive. So, what makes *x* – 1 = 0? The *x* value 1 makes the inside of the square root equal to zero. Any number bigger than this will make the square root positive.

Domain: $1 \leq x \leq \infty$

Range : $0 \leq y \leq \infty$

PRACTICE

Give the domain and range of the following tables.

1.

x	–2	–1	0	1	2
***f*(*x*)**	3	1	2	0	4

D ____________

R ____________

2.

x	0	1	3	4	9
***f*(*x*)**	–2	–1	1	5	2

D ____________

R ____________

3.

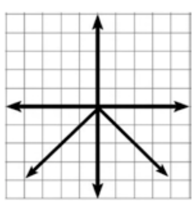
x	3	1	2	–3	–4
***f*(*x*)**	1	1	2	3	7

D ____________

R ____________

Give the domain and range of each function sketched below.

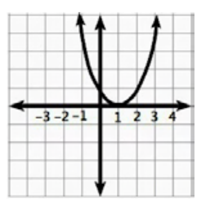
4.



D _________

R _________

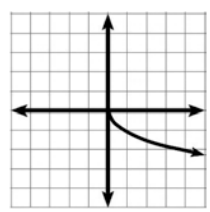
5.



D _________

R _________

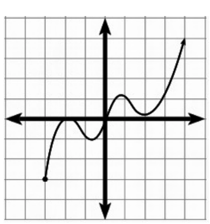
6.



D _________

R _________

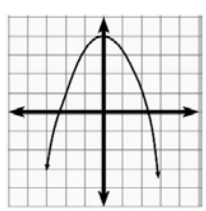
7.



D _________

R _________

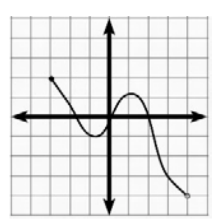
8.



D _________

R _________

9.



D _________

R _________

Give the domain and range of the following functions.

10. $f(x) = |x| + 6$ D: ______________ R: ________________

11. $f(x) = (x - 2)^2$ D: ______________ R: ________________

12. $f(x) = x^3 + 1$ D: ______________ R: ________________

13. $f(x) = \frac{1}{x+3}$ D: ______________ R: ________________

14. $f(x) = \sqrt{x+2}$ D: ______________ R: ________________

15. $f(x) = \sqrt{x-5}$ D: ______________ R: ________________

16. $f(x) = \sqrt{-x+1}$ D: ______________ R: ________________

ZEROS AND *X* AND *Y* INTERCEPTS

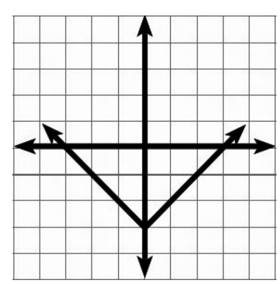
A **zero** of a function is any x value that makes the function equal to zero. On a graph, the zeros will also be the ***x*–intercepts**.

The ***y*–intercept** can be found by substituting zero into the function for x and evaluating.

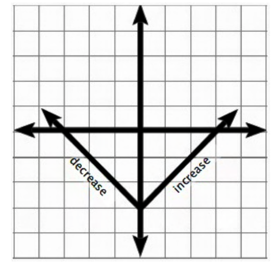
➤**Find the zeros, *x*–intercepts and *y*–intercept of the function $f(x) = 3x - 6$.**
Answer
The zero is the x–intercept. So, determine what makes $3x - 6 = 0$. Adding 6 to each side gives you $3x = 6$. Divide by 3 and the equation becomes $x = 2$. So, the zero is 2. The coordinate of the x–intercept is (2, 0).
Put 0 in for the x value to get the y–intercept. Evaluate 3(0) – 6. This gives –6. So the y–intercept is located at (0, –6)

➤**Find the zeros, *x*–intercepts, and *y*–intercept of $f(x) = 5x + 10$.**
Answer
For the zero and x–intercept: Set the function equal to zero. Now it is $5x + 10 = 0$. Then subtract 10 from each side. Now it is $5x = -10$. Then divide by 5. Now it is $x = -2$. So, the zero is –2 and the x–intercept is at (–2, 0).
For the y–intercept: Evaluate 5(0) + 10 and you get 10. The y–intercept is (0, 10).

➤**Use the sketch below to find the zeros, *x*–intercepts, and *y*–intercept.**
Answer
The zeros are given by the x–intercepts. The function intersects the x-axis at –3 and 3. So, the zeros are –3 and 3. The coordinates of the x–intercepts are (–3, 0) and (3, 0).
The y–intercept is at (0, –3)

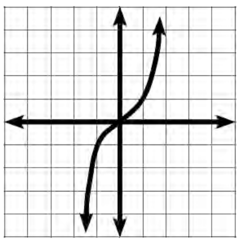


INTERVALS OF INCREASE AND DECREASE



To identify intervals of increase and decrease, a graph can be used. As you follow the function left to right, if the function is going down, it is decreasing. If the function is going up, it is increasing. When describing the intervals, make sure the *x* values are used.

Example
The function is decreasing along the *x* values from $-\infty$ to 0. The function is increasing from *x* values 0 to ∞. The point where the function changes from increasing to decreasing, the point is called an **extrema**. In this case, the point is a minimum point.



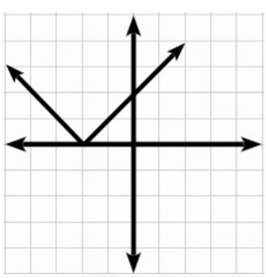
END BEHAVIOR

The end behavior of a function describes what the function does as the *x* values increase to very large positive values and very large negative values. For the absolute value function, as the "*x* values approach infinity," the *y* values approach infinity. The graph keeps going up and to the right. As the *x* values approach $-\infty$, the *y* values go up towards infinity.

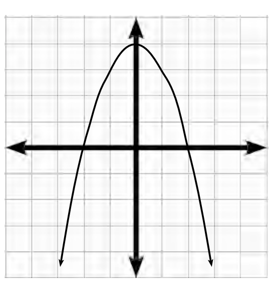
The function above goes up as the *x* values go out to the right. So, we say "as *x* is approaching infinity, the *y* values approach infinity." The function goes down and to the left. So as the *x* values approach negative infinity, the *y* values go down to negative infinity.

PRACTICE

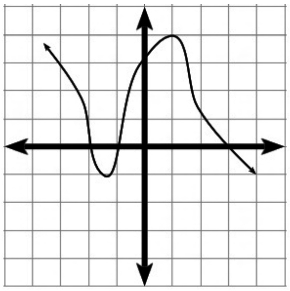
Given the graph of a function below, determine the characteristics based on the sketch.



Example
domain ___*all real numbers*___
range ___$y \geq 0$___
zeros ___*zero at* -2___
x-intercepts ___$(-2,0)$___
y-intercept ___$(0,2)$___
extrema ___*min at* $(-2,0)$___
interval of increase ___$-2 \leq x < \infty$___
interval of decrease ___$-\infty < x < -2$___
end behavior ___*as x approaches* ∞*, f(x) approaches* ∞___
___*as x approaches* $-\infty$*, f(x) approaches* ∞___



17. domain ______________________
range ______________________
zeros ______________________
x-intercepts ______________________
y-intercept ______________________
extrema ______________________
interval of increase ______________________
interval of decrease ______________________
end behavior ______________________
____



18. domain________________________
range________________________
zeros________________________
x–intercepts___________________
y–intercept___________________________
extrema______________________
interval of increase___________________
interval of decrease__________________
end behavior ________________________________
____

Given the functions below, determine the characteristics.

19. $f(x) = (x - 1)^2$
domain______________________________
range______________________________
extrema____________________________
interval of increase___________
interval of decrease__________
end behavior________________

20. $f(x) = -|x| + 2$
domain______________________________
range______________________________
extrema____________________________
interval of increase_________
interval of decrease__________
end behavior________________

21. $f(x) = (x + 4)^3 - 1$
domain______________________________
range______________________________
extrema____________________________
interval of increase___________
interval of decrease__________
end behavior________________

22. $f(x) = \sqrt{x - 1}$
domain______________________________
range______________________________
extrema____________________________
interval of increase_________
interval of decrease__________
end behavior________________

23. $f(x) = 4x^2$
domain______________________________
range______________________________
extrema____________________________
interval of increase___________
interval of decrease__________
end behavior________________

24. $f(x) = 3x - 12$
domain______________________________
range______________________________
extrema____________________________
interval of increase_________
interval of decrease__________
end behavior________________

25. $f(x) = 5x + 15$
domain______________________________
range______________________________
extrema____________________________
interval of increase___________
interval of decrease__________
end behavior________________

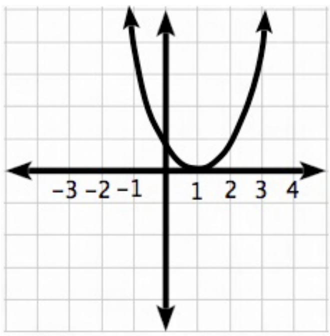
26. $f(x) = |2x - 8| + 2$
domain______________________________
range______________________________
extrema____________________________
interval of increase_________
interval of decrease__________
end behavior________________

27. $f(x) = (x - 4)^2 + 3$
domain______________________________
range______________________________
extrema____________________________
interval of increase___________
interval of decrease__________
end behavior________________

28. $f(x) = -3x - 1$
domain______________________________
range______________________________
extrema____________________________
interval of increase_________
interval of decrease__________
end behavior________________

CHAPTER 2 TEST

Use the function sketched below to answer questions 1–5.



1. What is the domain of this function?
   A. $x > 1$ B. $x = 0$
   C. $x < 1$ D. all real numbers

2. What is the range of the function?
   A. $y > 0$ B. $y = 0$
   C. $y < 0$ D. all real numbers

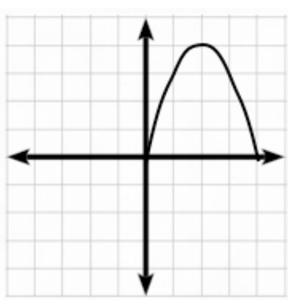
3. For what values is the function increasing?
   A. $x \geq 1$ B. $x \leq 1$
   C. $x = 1$ D. all real numbers

4. Where is an extrema located?
   A. (1, 1) B. (1, 0)
   C. (0, 1) D. (0, 0)

5. Where is the y-intercept of this function?
   A. (1, 1) B. (1, 0)
   C. (0, 1) D. (0, 0)

Use the following information for questions 6 and 7.

A toy water rocket is launched upwards. The sketch (right) represents the height of the rocket (y) in meters over time (x) in seconds.



6. What do the zeros represent in this case?
   A. the highest point the rocket reaches
   B. the time when the rocket is on the ground
   C. the time when the rocket is moving up
   D. the time when the rocket is moving down

7. At what time is the rocket at its highest point?
   A. 1 second B. 2 seconds
   C. 3 seconds D. 4 seconds

8. What is the domain of the function $f(x) = \sqrt{x - 5}$?
   A. all real numbers
   B. all $x \geq 5$
   C. all $x \leq 5$
   D. all $x \geq -5$

9. Finish the statement. For $f(x) = x^3$, as x approaches ∞, $f(x)$ approaches
   A. ∞. B. $-\infty$.
   C. 0. D. 3.

10. What is the range of the function $f(x) = -|x| - 1$?
   A. $y \leq -1$ B. $y \leq 1$
   C. $y \geq 1$ D. $y \geq -1$